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How to model your mortality/longevity risks?

Focus on Taiwan mortality data

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Contents



- Introduction
- Stochastic mortality modeling
- Case study 1 Pricing an annuity product
- Case study 2 Mortality / longevity stress test
- Bibliography





Introduction Aging population (1/3)

• The projected population pyramids are shifting upward with thinner waist in 2050 compared to 2010.



Introduction Aging population (2/3)



That follows the general public is facing a longer life than previous generations.



Life expectancy (Male)

Introduction Aging population (3/3)





Life expectancy (Female)

Source: United Nations, Population Division Department of Statistics, MOI, Taiwan







Introduction Old age dependency ratio is increasing





Introduction

Projecting future longevity: questions and debates

- No consensus among demographers, academics and actuaries regarding future mortality
 - Impact of medical advances? Any biological limit to life expectancy?
 - Acceleration or deceleration of mortality improvements?
 - What happened in the past can happen in the future
- Debating regarding the mortality data
 - Assured data vs national population data
- Debate regarding the projection models to use
 - Extrapolative models (those developed in this presentation)
 - Cause-specific models



Introduction Mortality risks

- Different mortality risks:
 - catastrophic risk -> pandemics, terrorism, ...
 - mortality risk except CAT, split into :
 - a trend risk : the extension of life expectancy (longevity risk)
 - oscillations around a trend
 - Small size / non homogeneous structure of an insurance portfolio





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Introduction Definition

Mortality rate for a given generation

- Notations and indicators : ۲
 - Mortality rate at the age of x and at year t: q(x,t)
 - -> probability of dying during year t for a person aged of x at the beginning of the year
 - Number of deaths at the age of x and at year t: D(x,t)
 - Instantaneous rate of mortality whose age is x and during year t: $\mu(x,t)$
 - -> probability (assumed constant during the year) of dying instantaneously for a person x year old

2002 2003 2004 2005 2006 2007 Age / Year 2008 0.010748 0.010551 0.01233 0.01251 0.01177 0.01186 0.010948 60 0.01341 0.01328 0.01318 0.01258 0.011943 0.011737 61 0.012152 62 0.01472 0.01395 0.01446 0:013133 0.01496 0.0129 0.013369 0.014391 63 0.01605 0.01573 0.01533 0.01511 0.014141 0.013896 0.01746 0.0171 0.0165 64 0.01653 0.015602 0.015336 0.015074 Mortality 65 0.01881 > 0.01852 0.01759 0.01784 0.016874 0.016577 0.016285 **Observed mortality Projected mortality** Mortality rate of American men – source HMD



rate for given year and age





Introduction Analysis



• Analysis of mortality rates through 3 main factors:



Sources : Cairns J. G.et al., 2007, A quantitative comparison of stochastic mortality models using data from England & Wales and the United States





Introduction Implications (1/2)



- Pricing / EV implications. Companies usually use flat percentages of the regulatory mortality tables as mortality assumptions for the purposes of pricing / valuation. Using a mortality model helps to better assess the future mortality rates of an insurance portfolio.
- **Capital requirement / internal models / stress tests analysis**. Modelling mortality risks allows to integrate the mortality scenarios within an internal model / or calibrate mortality stress tests more in line with the specificities of the country / insurance company.
- Capital market solutions. Modelling mortality is a must to price / understand the risk of securitization products like Mortality CAT bonds (catastrophic mortality risk), longevity bonds (longevity risk), longevity swaps (longevity risk current market activity focused on longevity swaps).



Introduction Implications (2/2)



AXA France and Hannover Re announce the conclusion of the first longevity swap on the French market

Hannover, 8 August 2014:

The reinsurance treaty covers more than 22,000 insured with associated pension liabilities which represent nearly EUR 750 million.

Source: Hannover Re website



Contents



- Introduction
- Stochastic mortality modeling
- Case study 1 Pricing an annuity product
- Case study 2 Mortality / longevity stress test





Cohort effect

Stochastic mortality modeling Overview (1/2)

- Models rely on a dynamic of instantaneous mortality rates m(x,t) or of annual mortality rates q(x,t)
- Modeling instantaneous mortality rate

-Lee-Carter (1992)/ Brouhns Denuit (2002)

-Renshaw-Haberman (2006)

-Currie Age-Period-Cohort (2006)

Modeling mortality rate

-CBD2 *(2007)*

-CBD3 (2007)

 $logit \ q(x,t) = \ln\left(\frac{q(x,t)}{1-q(x,t)}\right) = k_t^{(1)} + k_t^{(2)}(x-\bar{x}) + k_t^{(3)}((x-\bar{x})^2 - \sigma_x^2) + \gamma_{t-x} + \log t \ q(x,t) = k_t^{(1)} + k_t^{(2)}(x-\bar{x}) + \gamma_{t-x} (x_c - x)$

Pure age

 $\ln \mu(x,t) = \beta_x^{(1)} + \beta_x^{(2)} k_t + \beta_x^{(3)} \gamma_{t-x}$ $\ln \mu(x,t) = \beta_x + k_t + \gamma_{t-x}$

effect

 $\ln \mu(x,t) = \beta_{x}^{(1)} + \beta_{x}^{(2)}k_{t}$

• **P-Splines (2004)** $\ln \mu(x,t) = \sum_{i,j} \theta_{i,j} B_{i,j}^{ay}(x,t)$



Period effect

Stochastic mortality modeling Overview (2/2)



- Other approaches have been developed more recently such as Luciano-Vigna 's model, Gourieroux-Monfort's model
- Key idea: create a risk-neutral measure based on both market risk and demographic risk
- Transposition of the interest rate theory to mortality modeling:

Modeling the mortality	Modeling the interest rate
Instantaneous mortality rate for a person at the age of x at time s: $\mu(x,s)$	Instantaneous interest rate at time s: r(s)
Probability to be alive in <i>t</i> for an insured person with age of <i>x</i> in 0: ${}_{t} p_{x} = E \left[\exp \left(-\int_{0}^{t} \mu(x+s,s) ds \right) \right]$	Price in 0 of a t-maturity bond: $P(0,t) = E\left[\exp\left(-\int_{0}^{t} r(s)ds\right)\right]$



Stochastic mortality modeling Calibration (1/3)



• Example - Brouhns-Denuit model: $\ln \mu(x,t) = \beta_x^{(1)} + \beta_x^{(2)}k_t$



Step 1 :

Inputs based on national population data (HMD database)

Step 2 : Calibration of the model parameters by maximum likelihood





Stochastic mortality modeling Calibration (2/3)



• Example (cont'd) - Brouhns-Denuit model : $\ln \mu(x,t) = \beta_x^{(1)} + \beta_x^{(2)}k_t$



Step 3 :

Extrapolation of the time series k_t using a process ARIMA(0,1,0)



Step 4 : *Simulation of the probability of death*





Stochastic mortality modeling Calibration (3/3)



• Illustration : estimation of the Brouhns-Denuit model on Taiwanese, Japanese, Australian and UK data:







Stochastic mortality modeling Models comparison (1/5)

Key criteria to compare mortality models

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 Reasonableness of future projected mortality rates (back-testing / comparison with publicly available forecasts)



Ability to generate distribution of future mortality rates



Check key statistical properties / reasonable pattern of standardised residuals



Ability to capture the cohort effect



Computation time (calibration process + simulation process)



Parsimony (models with few parameters are recommended)



Robustness of parameter estimates

Some of these criteria can be conflictual (e.g. ability to capture cohort effect versus parsimony)

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Stochastic mortality modeling Models comparison (2/5)









Stochastic mortality modeling Models comparison (3/5)









Stochastic mortality modeling Models comparison (4/5)



Key criteria to compare mortality models



Reasonableness of future projected mortality ra

Distribution of mortality rate



Statistical properties

Cohort



Computation time



Parsimony



1. Different sets of ages (base case age 40 to 80) -3 Parameter Age effect Impact on 50 53 56 59 62 65 estimated parameters 20-100 50-70 -7 40-80 Projected BE montality rafes % 60 years old 40-80 Impact on 20-100 projected mortality rates 50-70 Y1 Y2 Y10 Y3 Y4 **Y**5 Y6 Y7 ¥9 Note: Black line is the 5% confident interval for 40-80 2. Different periods (base case 1970-2010) 0 Age Pagameter - Age 50 53 56 59 62 65 68 Impact on effect estimated parameters 1970 - 2010 1980 - 2010 2000 - 2010 -8 60 years old Projected BE 0.2% 0.7% 0.7% 1970-2010 Impact on projected 1980-2010 mortality rates 2000-Y1 Y2 Y10 Y3 Y4 2010 Note: Black line is the 5% confident interval for 40-8 🕻 Milliman

Estimation of Lee Carter model using different sets of

historical data:



Stochastic mortality modeling Models comparison (5/5)



• P-Splines (2004) $\ln \mu(x,t) = \sum_{i,j} \theta_{i,j} B_{i,j}^{ay}(x,t)$



Contents



- Introduction
- Stochastic mortality modeling
- Case study 1 Pricing an annuity product
- Case study 2 Mortality / longevity stress test



Annuity business in Asia Taiwan





- Fixed annuity and variable annuity account for 25% and 1% of the market in terms of new business premium in 2013.
- The most popular product is interest sensitive deferred annuity which competes with other deposit replacement type products.
- The pricing of traditional payout annuity is difficult due to long stagnantly low interest rate and the lack of tax benefit in retirement plans.
- The government has initiated enterprise annuity scheme since 2005 but the minimum guarantee of 2-year CD rate is considered too stringent and thus not many players are eager in this market so far. But it is likely that the government will allow more asset types for investment under annuity insurance scheme, even risky assets.



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Source: Taiwan Life Insurance Association, 2013

Annuity business in Asia





- Japan is the world's second largest variable annuity market with ¥18 trillion of assets under management as at 2012. But the variable annuity market is struggling to recover its pre-crisis momentum.
- Despite of the asset deflation and the extremely low interest rates, the general public is reluctant to put their savings at risk into the capital market considering the remaining shaky market. It naturally follows that the aging population is moving towards investment with principal guarantees with some potentially upside gains.



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Source: Life Insurance Association of Japan, 2012





Annuity business in Asia Singapore and Hong Kong



Source: Life Insurance Association of Hong Kong, 2012; Life Insurance Association of Singapore, 2013





Annuity business in Asia China and South Korea



China

- The Enterprise Annuity Fund Management Regulation was initiated on May 1, 2011, to ensure sustainable performance of enterprise annuities and to devise a framework to regulate fund investments.
- Ping An Annuity is the leading provider of enterprise annuities, and is also the only pension insurer with a nationwide network, with over RMB 60 billion in investment assets in 2012.
- The government has been in the process of devising the taxdeferred policy for annuity products.

South Korea

- Variable annuity is continuously growing while fixed annuity is rather silent in the market.
- Despite a short history since the introduction of VA, most products offered contain ancillary benefits (GMXBs) with occasional high benefit designs for which the major risks are interest rate risk, equity risk...etc but longevity risk is rather insignificant considering the product designs.







Annuity business in Asia Presence of Longevity risk

- In these markets just discussed, the annuity products generally account for a relatively small part of the market (except Japan) or the longevity risk is not significant given the product features (e.g. deferred annuities with small annuitization probabilities). This is mainly because
 - the interest rates have remained low for the past decade, hence it is difficult to have an attractive annuity products in these markets;
 - lack of incentives to enter annuity markets due to regulation constraints
 - investors or policyholders generally prefer deposit replacement type of products.
 - it is difficult and expensive to offer attractive guarantees in annuity products, making it less competitive.
- But there are some initiatives to offer enterprise annuities under the public pension schemes. We expect the longevity risk may gradually become an issue with aging population and more people looking for income protection towards retirement.





Pricing an annuity product Base case (1/3)



Consider an old immediate annuity that was priced based on 95TIA

The immediate annuity receives GP in exchanges for a series of monthly annuity payment till death, subjected to the guaranteed amount (GP)



Pricing Assumption			
Mortality	100% 95TIA		
Withdrawal Rate	0%		
General operating expense	NTD 1,100 upfront		
Pricing interest rate	4% level		
Guaranteed interest rate	3% level		
Commission	1%		





Pricing an annuity product (Cont'd) Base case (2/3)



- The best estimate mortality implied by the stochastic model is lower.
 - The original pricing mortality table is 95TIA, but the best estimate simulated from stochastic mortality model is much lower.
 - The ratio of stochastic mortality rate to 95TIA may drop as much as to 40% of 95TIA.





Pricing an annuity product (Cont'd) Base case (3/3)



• By applying the best estimate mortality rates from Lee-Carter model, the profit margin significantly drops.

Profit margin by issue age and gender					
Male	Original pricing	Stochastic mortality rates	Female	Original pricing	Stochastic mortality rates
20	16.1%	13.5%	20	16.7%	14.2%
30	14.5%	11.2%	30	15.1%	11.9%
40	12.6%	8.6%	40	13.3%	9.3%
50	10.4%	6.0%	50	11.1%	6.6%
60	8.0%	3.9%	60	8.6%	4.3%
70	5.6%	2.7%	70	6.1%	2.9%





Pricing an annuity product (Cont'd) Sensitivity (1/2)



Mortality rates

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- The aging population has long become an issue in Japan, with more experience accumulated regarding to mortality improvement.
- If the Taiwan experience followed that Japan, let's test the impact on the annuity profit margin.





Pricing an annuity product (Cont'd) Sensitivity (2/2)



• Sensitivity result if the Taiwan mortality improvement trends followed that of Japan

Profit margin by issue age and gender							
Male	Original pricing	Stochastic mortality rates – Slope follows Taiwan	Stochastic mortality rates – Slope follows Japan	Female	Original pricing	Stochastic mortality rates – Slope follows Taiwan	Stochastic mortality rates – Slope follows Japan
20	16.1%	13.5%	12.9%	20	16.7%	14.2%	13.4%
30	14.5%	11.2%	10.5%	30	15.1%	11.9%	10.7%
40	12.6%	8.6%	7.9%	40	13.3%	9.3%	7.6%
50	10.4%	6.0%	5.3%	50	11.1%	6.6%	4.3%
60	8.0%	3.9%	3.0%	60 1	8.6%	4.3%	1.5%
70	5.6%	2.7%	1.7%	70	6.1%	2.9%	0.2%





Contents



- Introduction
- Stochastic mortality modeling
- Case study 1 Pricing an annuity product
- Case study 2 Mortality / longevity stress test
- Bibliography



Mortality / longevity stress tests Introduction



• Solvency II and RBC frameworks provide standardized stress tests to apply on mortality rates

Capital requirement	Risk	Stress test based on SII standard formula
Life _{mort} =∆NAV mortality stress	Mortality risk	A permanent 15% increase in mortality rates for each age and each policy where the payment of benefits is contingent on mortality risk
Life _{long} =∆NAV longevity stress	Longevity risk	A permanent 20% decrease on mortality rates for each age and each policy where the payment of benefits is contingent on longevity risk
Life _{CAT} = Δ NAV Life catastrophe stress	Life catastrophe risk	Absolute increase in the rate of policyholders dying over the following year of 1.5 per mille

 Using a stochastic mortality model can be useful to assess a more country specific mortality / longevity set of stress tests:





Mortality / longevity stress tests Proposed approach (1/4)





Mortality / longevity stress tests Proposed approach (2/4)

• How to project the best estimate stress scenario?...



... Using one of the different stochastic mortality models shown in the previous slides





Mortality / longevity stress tests Proposed approach (3/4)

• How to project the stress scenario?



Projection of an adverse scenario using a stochastic mortality model (e.g. 0.5 percentile of the mortality distribution)

Project the future mortality rates conditionally to the first period adverse scenario and then deduce the conditional best estimate mortality rates (see the trend component of the Lee Carter model)





Mortality / longevity stress tests Proposed approach (4/4)





$$\begin{array}{c} \mathbf{x} = \\ \{\mathbf{x}_{\min}, \, \dots, \, \mathbf{x}_{\max}\} \end{array} \quad \Delta_{x,1} = \frac{q^{Choc,\varepsilon}(x,1) - \frac{1}{S} \sum\limits_{s=1}^{S} q^{s}(x,1)}{\frac{1}{S} \sum\limits_{s=1}^{S} q^{s}(x,1)} \\ \left\{ \mathbf{x}_{\min}, \, \dots, \, \mathbf{x}_{\max} \right\} \end{array} \quad \Delta_{x,1} = \frac{q^{Choc,\varepsilon}(x,1) - \frac{1}{S} \sum\limits_{s=1}^{S} q^{s}(x,1)}{\frac{1}{S} \sum\limits_{s=1}^{S} q^{s}(x,1)} \\ \Delta_{x,t} = \frac{\frac{1}{S} \sum\limits_{s=1}^{S} \left\{ q(x,t) \middle| q^{Shock}(x,1) \right\} - \frac{1}{S} \sum\limits_{s=1}^{S} q^{s}(x,t)}{\frac{1}{S} \sum\limits_{s=1}^{S} q^{s}(x,t)} \\ \end{array}$$







Case study 2 – Mortality / longevity stress tests Results (1/2)

• Stress tests - Illustration on Taiwan total population (Male + Female):





Case study 2 – Mortality / longevity stress tests Results (2/2)

- CEIOPS, Consultation paper No. 49 « Standard formula SCR Article 109 c Life underwriting risk » (2009)
 - «

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- 3.42 Subsequent to QIS4, an investigation has been carried out by the Polish FSA which analysed the mortality data for nine countries indicated based both on historic improvements and a stochastic model of future mortality improvements.
- 3.43 "The results of this analysis indicated that, on average (across the nine countries for which data was analysed), historic improvements in mortality rates over 15 years from 1992 to 2006 were higher than 25%. Although the results of the stochastic model of future mortality improvements may imply a lower stress, CEIOPS has attached more weight to the analysis of historic improvements because of the significant uncertainty inherent in modelling mortality



Contents



- Introduction
- Stochastic mortality modeling
- Case study 1 Pricing an annuity product
- Case study 2 Mortality / longevity stress test
- Bibliography



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